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Teaching about Gauss's Law by Combining Analogical and Extreme Case Reasoning

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ABSTRACT

It is well known that many students have tremendous difficulties with applying Gauss's law for purposes of solving quantitative as well as qualitative problems. In this study, it was investigated how understanding of Gauss's law can be facilitated by analyzing the superposition of electric field vectors for increasingly complex geometric configurations of charges. Actually, in the spring semester of academic year 2016/2017, a pretest-posttest quasi-experiment was performed with 180 students from the Faculty of Chemical Engineering and Technology in Zagreb, Croatia. The student sample has been divided into three control subgroups and three experimental subgroups. Control subgroups (Nc=93) received a traditional teaching treatment while in the experimental subgroups (Ne=87) students showed how reasoning about superposition of electric field vectors can be transferred from relatively simple configurations of charges to more complex ones. At the posttest, students from the experimental group proved to be significantly more effective in solving qualitative problems on Gauss's law. The results from our study support the idea that development of analogical, visually rich models facilitates the meaningful learning.

Keywords: Gauss's law, superposition of electric field, analogies, extreme case reasoning.

INTRODUCTION

Electricity has intrigued people since ancient times. As a matter of fact, people reasoned about the nature of lightning (Viegas, 2007), conducted experiments with static electricity (Stewart, 2001) and reported about some interesting electrical phenomena in the animal world (Moller, 1991) even long before adopting the modern understanding of electricity. However, it took many centuries until the development of a more coherent view about electricity. Significant contributions to a better understanding of electrical phenomena have been made by Carl Friedrich Gauss. Specifically, Gauss came to the conclusion that the net electric flux through an arbitrary closed surface is proportional to the net electric charge within that closed surface regardless of electric charge distribution and shape of the surface. This conclusion has been known as Gauss's law and is one of the four Maxwell equations that form the basis of classical electrodynamics theory (Jackson, 1999). The importance of Gauss's law particularly

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stems from the fact that it can be used to calculate the electric field at arbitrary points. Thus, it can be concluded that students' ability to apply Gauss's law in qualitative and quantitative contexts represents an important requirement in typical electromagnetism curricula at the university level. However, past researches have shown that Gauss's law is a very challenging topic for most students (Aubrecht & Raduta, 2005; Guisasola, lmudi, Salinas, Zuza, & Ceberio, 2008; Maries, Lin, & Singh, 2017; Lin, Maries, & Singh, 2012; Pepper, Chasteen, Pollock, & Perkins, 2010; Singh, 2006). Concretely, it has been found that students in introductory physics courses often use already derived formulae to calculate the electric field without considering the assumptions about symmetry conditions (Traxler, Black, & Thompson, 2007). As a matter of fact, students often memorize formulae of the magnitude of electric field without paying attention to symmetry considerations and they have difficulties with identifying situations where Gauss's law is applicable (Singh, 2006).

Many of the identified difficulties are probably related to an abstract nature of electrostatic examples which are typically considered in lectures about Gauss's law (Demirci & Çirkinoglu, 2004; Isvan & Singh, 2006). Another possible source of students' difficulties with Gauss's law is related to the fact that many students have a relatively weak mathematical (integral calculus) background when they first introduced about Gauss's law (Grundmeier, Hansen, & Sousa, 2006). Concretely, Bollen, van Kampen and De Cock (2015) found that students often have poor understanding of operators, graphical representation of vector fields and have problems with conceptual interpretation of calculus even they know how to use. So, the requirement often results in cognitive overload in order to combine abstract physics concepts with relatively sophisticated mathematical tools. As a matter of fact, past studies showed that students have difficulties with understanding the concept of electric field as well as with applying integral calculus and it is no surprise that they are struggling with applying integral calculus for purposes of reasoning about an electric field (Bollen et al. 2015; Pepper, Chasteen, Pollock, & Perkins, 2012).

The reasoning is highly demanding and may result in cognitive overload since require combining multiple knowledge elements (Adams, 2015; Bloom, & Krathwohl, 1956; Sorden, 2005; Sweller, Van Merrienboer, & Pass, 1998). The probability of cognitive overload can be reduced by breaking down the lecture into smaller sections using effective external visualizations, providing explicit cues and making sure that students have a solid understanding of basic concepts/examples before considering more complex ones. Within the context of learning and teaching about Gauss's law, cognitive overload may be prevented by relating it to the principle of superposition of electric field for relatively simple geometrical configurations of point charges before going on with more complex configurations and continuous charge distributions. So, it is necessary that students have expertise in superposition of vectors which is a prerequisite for finding the net electric force/field at an arbitrary point in order to develop higher expertise in Gauss's law (Sing, 2006). However, earlier researches showed that students have substantial difficulties in understanding the principles of superposition and its application in tasks that involve the electric field concept (Bagno & Elyon, 1997; Bagno, Eylon, & Ganiel, 2000; Li & Singh, 2017; Viennot & Rainson, 1992). Indeed, Aubrecht (2005) found that for many students it is difficult to transfer their knowledge of the superposition principle from the context of mechanics (where it first introduced) to electromagnetism. At the same time, Singh (2006) pointed out that understanding of the principle of superposition of electric field is a prerequisite for effective usage of Gauss's law and instructional strategies should take into account in students' difficulties with the principle of superposition that are focused on improving student understanding of Gauss's law. In last, Bollen et al. (2015) suggested that teachers should put an additional effort to present more graphical and conceptual examples in order to explicitly relate the abstract equations to vivid electrical phenomena.

It has been generally recognized that there is a need for a teaching method which will foster students' ability to effectively apply the Gauss's law in qualitative and quantitative contexts (Bollen et al., 2015; Pepper et al., 2010). The ability to solve qualitative problems requires not only developing expertise in mathematics but also development of visually rich mental models (Greca & Moreira, 1997). On the other hand, development of visually rich mental models of physics phenomena is often associated with analogical and extreme case reasoning. So, analogical reasoning is characterized as transferring information or meaning from a particular source domain to a target domain (Maymoona & Sulaiman, 2015; Podolefsky & Finkelstein, 2006; Sevim, 2013). In addition, extreme case reasoning happens when a situation E (extreme case) is suggested in order to facilitate reasoning about a situation A (the target) in which some aspect of situation A has been maximized or minimized (Stephens & Clement, 2009). Using analogies and extreme cases was at the heart of some of the greatest discoveries in history of physics such as the inertia concept discovered by Galilei (Einstein & Infeld, 1938; Nersessian, 1999). When it comes to electromagnetism, it should be noted that Thompson developed formal analogies between electrostatics and heat flow while Maxwell drew analogies between movement of incompressible fluid and electrostatics (Silva, 2007). Past researches showed that using analogies and extreme cases may facilitate learning of abstract and counterintuitive concepts (Clement, 1991, 1993; Stephens & Clement, 2009, 2010; Zietsmann & Clement, 1997). This can be related to the fact that analogical and extreme case reasoning fosters the development of imaginable, intuitive, and grounded explanatory models (Clement, 1993). On the other hand, simultaneous reasoning about the source and target domain can result in cognitive overload in ill-designed analogy-based instruction (Johnstone & Al-Naeme, 1991; Lin & Chiu, 2017). In order to optimize the cognitive load, researchers have suggested strategies such as breaking down the lecture into smaller "chunks" and providing explicit clues as well as using external visualizations and personally familiar analogical anchors (Van Merrienboer & Sweller, 2005).

In this study, it is purposed to present how reasoning about net electric field (at an arbitrary point) for an one-dimensional configuration of point charges (source domain) can be gradually transferred to more complex contexts that require reasoning about net electric field (at an arbitrary point) that results from a three-dimensional, continuous distribution of charges (target domain). Concretely, in our experimental teaching intervention, equidistant charges were used on a straight line as an anchoring example and showed how to apply the superposition principle in that context. Thereafter, the superposition principle has been applied for charges fixed in vertices of a square and octagon as well as to a circle as an extreme case of a polygon when the number of edges tends to infinity. Finally, it was showed that how reasoning about superposition from the context of uniformly charged circles can be transferred to the context of charged spheres. Thereby, the volume of a charged sphere has been conceptualized as an extreme case of a surface charged spherical shell – it can be thought as consisting of a large number of concentric, surface charged shells of varying radii.

Purpose

In this study, it was aimed to investigate whether analogies and extreme cases used teaching intervention can foster students' ability to apply Gauss's law in qualitative and quantitative contexts. Indeed, analogies were drawn between applying the principle of superposition for 1D/2D configurations of charges (source domain) and 3D configurations of charges (target domain), and a volume charged sphere has been conceptualized as an extreme case of a surface charged sphere.

The significance of this study is related to the fact that it provides an example of how developing visually rich, analogical models can foster the students' ability to transfer their knowledge to new contexts.

DESIGN AND METHODS

a) Research design

In this study, a pretest-posttest quasi-experiment was implemented with the aim to investigate whether enriching traditional teaching with analogical and extreme case reasoning can help the students to develop a higher ability for solving qualitative and quantitative problems within the context of Gauss's law. The student sample has been divided into three control and three experimental subgroups. The experimental subgroups received a teaching treatment enriched by analogies and extreme cases, while the three control subgroups received traditional practice sessions in which most of the time the teaching assistant modeled solving of quantitative problems. Both treatments lasted for 2 teaching hours (90 minutes). All subgroups took the pretest one week before the teaching treatment and posttest right after the treatment. The time for finishing the pretest and posttest was set to 15 and 20 minutes, respectively.

b) Participants and curriculum

Our sample included 180 first year students who were enrolled in the introductory physics course at the Faculty of Chemical Engineering and Technology in Zagreb, Croatia. The sample was divided into 6 subgroups whereby 3 subgroups received the control treatment and the remaining three subgroups received the experimental treatment.

In our sample, most of the students were 19-year olds and before their university education they had finished the eight-year primary and four-year secondary education. After graduating from the five-year study program at Faculty of Chemical Engineering and Technology, they can apply for jobs in industry or they can continue their education at PhD level. It should be noted that 73% of students in our sample were females and gender distribution was similar across all subgroups.

In Croatia, at the primary school level, students only learn the most basic facts about electrostatics such as learning to differentiate between positive and negative charges as well as a qualitative formulation of the Coulomb's law. At the secondary school level, this knowledge is broadened and deepened through student learning about the electric field, quantitative form of Coulomb's law as well as about electrostatic potential and work in the electrostatic field. However, Gauss's law is taught at the secondary school only at the level of conceptual explanations and simpler mathematics while at the university level students are applying the Gauss's law in many different (mainly quantitative) contexts due to its relatively complex mathematical representation. Generally, the curriculum of the introductory physics course at Faculty of Chemical Engineering and Technology can be characterized as a typical introductory physics course for scientists and engineers which follows standard textbooks such as Physics for Scientist and Engineers by Serway & Faughn (2006). It includes 2 hours of theoretical and 2 hours of practice sessions per week in both semesters of the first year of university. The theoretical sessions follow a traditional approach in which emphasis is on learning factual knowledge, whereas in practice sessions (Redish, 2003) the emphasis is on solving quantitative physics problems.

c) Treatment

Our study has been conducted in the spring semester of academic year 2016-2017 and it has been situated within the regular curriculum. Students from all subgroups received the same traditional theoretical sessions about electric field phenomena. Practice sessions for all students lasted in the same time (90 minutes) and were conducted by the same teaching assistant who is also the first author of this article. At the time of the experiment, the teaching assistant had five years of teaching experience.

In all subgroups we covered the same concepts related to electric field and Gauss's law.

In practice sessions, students from control subgroups received a typical traditional treatment characterized by solving of quantitative problems. Actually, the teaching assistant modeled the problem solving on blackboard and asked students to participate in discussion of most important aspects of the problem solving process. A brief description of quantitative problems that were solved and discussed in the control subgroups is given in Table I.

For the experimental subgroups, the traditional practice sessions have been enriched by visual presentation of analogies and extreme cases. So, the teaching assistant solved one quantitative problem less than in the control subgroups during the experimental treatment (Table I). The students from experimental subgroups were showed visualizations of the electric field for different geometric configurations of charges prior to solving quantitative problems. Firstly, the students were presented with a visualization of the electric field of an isolated positive/negative point charge and they were asked how the direction of field lines is related to the electric field vector. Then, they were asked to predict what happens if there is a two point charges as well as to relate this situation to the superposition principle. In last, students were showed an array of equidistant, positive charges that were fixed on a straight line (Figure 1) and they were asked to discuss the magnitude of electric force that acts on the individual charges. From symmetry considerations and superposition principle, students easily concluded the electric force on the central charge amounted to zero and that the largest force acted on the charges at the left and right end of the straight line. Since this one-dimensional configuration of positive charges proved to be easily comprehensible for most students, it was used as a cognitive anchor for developing understanding about more complex situations. In the next step, the teaching assistant showed a square frame with positive charges in its four vertices. Again, combination of symmetry considerations and principle of superposition helped the students to realize that the net electric field at center of the square was zero and the same conclusion has been drawn for an octagonal frame (Figure 1). In last, students were required to engage in extreme case reasoning in other words to predict what would happen if the number of vertices of the polygon tends to infinity. Most students realized that there is a uniformly charged (linear charge density λ=const) circle and the electric field at the center of the circle will be zero. Then, it was aimed to qualitatively discuss what happens if we move to an arbitrary point P located inside the circle, but right to the center of the circle (Figure 2). It was clear that the ratio of lengths l_1 and l_2 of circular arcs at opposite sides of point P is the same as the ratio of r_1 and r_2 in Figure 2. Thus, the ratio of net electric charges (which are proportional to length of the arcs) for these two circular arcs also amounts to r_1/r_2. By taking into account that the electric field is proportional to q/r^2, it follows that the electric field from the circular arc that is right from P has a larger magnitude in P than the electric field from the circular arc located left to P. In a nutshell, the field in P is different from zero because the effect of larger amount of charge on left circular arc is not exactly balanced by the smaller distance of right circular arc from point P. As a matter of fact, since $E_j = k\frac{q_j}{r_j^2}, j = \mathrm{L}, \mathrm{R}$ and $q_j = \lambda l_j = \lambda r_j \theta$ it follows:

$$E_L = k\frac{q_\mathrm{L}}{r_\mathrm{L}^2} = \frac{\lambda r_\mathrm{L}\theta}{r_\mathrm{L}^2}$$

$$E_R = k\frac{q_\mathrm{R}}{r_\mathrm{R}^2} = \frac{\lambda r_\mathrm{R}\theta}{r_\mathrm{R}^2}$$

$$\frac{E_L}{E_R} = \frac{\frac{\lambda r_L\theta}{r_L^2}}{\frac{\lambda r_R\theta}{r_R^2}} = \frac{r_R}{r_L}$$

Finally, the purpose was to explain what happens in the case of a three-dimensional object. Indeed, a sphere was considered (Figure 3) whose surface was uniformly charged (surface charge density σ=const). From symmetry considerations, it was easily come to the conclusion that both halves of the sphere have the same charge as well as average distance to the center of the sphere. Thus, on average the field vectors from the both semi-spheres have equal magnitude but opposite directions which means that the net field in the center of sphere is zero. In last, there was an attempt to explore the electric field outside the center of a surface charged sphere by using an approach analogous to the one we have used in the already considered as "charged circle". Thereby, an analogue to the plane angle θ subtended by a circular arc was the solid angle Ω (sphere) subtended by a part of spherical surface S. Then, for an arbitrary point inside the sphere, the superposition of net electric fields has been discussed due to parts of surfaces on opposite sides of that observed point. Indeed, since $q_j = \sigma S_j = \sigma r_j^2 \Omega$, $j = \mathrm{L}, \mathrm{R}$ it follows:

$$E_L = k\frac{q_{\mathrm{L}}}{r_{\mathrm{L}}^2} = \frac{\sigma r_{\mathrm{L}}^2 \Omega}{r_{\mathrm{L}}^2}$$

$$E_R = k\frac{q_{\mathrm{R}}}{r_{\mathrm{R}}^2} = \frac{\sigma r_{\mathrm{R}}^2 \Omega}{r_{\mathrm{R}}^2}$$

$$\frac{E_{\mathrm{L}}}{E_{\mathrm{R}}} = \frac{\frac{\sigma r_{\mathrm{L}}^2 \Omega}{r_{\mathrm{L}}^2}}{\frac{\sigma r_{\mathrm{R}}^2 \Omega}{r_{\mathrm{R}}^2}} = 1$$

Thus, superposition principle can be used to make the statement about zero field inside metal conductors which are more intuitive.

Only a brief, qualitative description has been provided for volume charged spheres. It was emphasized that volume charged spheres can be considered as extreme cases of surface charged spheres. As a matter of fact, the sphere can be considered for volume charged spheres as consisting of a large number of thin, concentric spherical shells of varying radii. If the field was considered at an arbitrary point P, there is exactly one shell X that passes through that point (Figure 3). The electric fields from the shells with radii greater than radius of shell X eliminate each other at P in a similar way as already described for surface charged spheres. However, the contributions of shells with radii smaller than radius of shell X do not eliminate each other. Symmetry arguments can be used to show that the center of charge for these inner shells is located at the center of the volume charged sphere which results with the fact that generally the electric field inside a volume charged sphere is not zero and increases with increasing distance from the center of the sphere.

Table 1. *A brief description of quantitative problems that were solved in the practice sessions. An asterisk denotes problems that were solved only in control subgroups.*

Problem 1	Calculating the coordinates of the location where total force on charge which is between two other fixed charges is zero.	Open-ended
Problem 2	Calculating electric field magnitude of the points inside and outside of a surface charged sphere.	Open-ended
Problem 3	Calculating the magnitude of electric field for points inside and outside of a volume charged sphere.	Open-ended
Problem 4	Calculating magnitude of the electric field at the point located at distance *r* from infinite cylindrical conductor.	Open-ended
*Problem 5	Calculating the surface charge density of a hollow sphere and magnitude of electric field at the surface of the sphere.	Open-ended

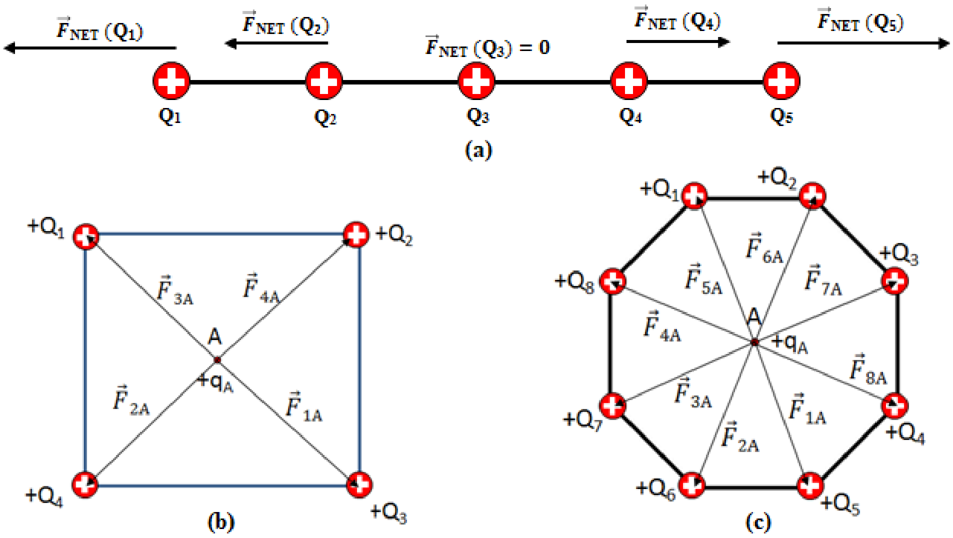

Fig. 1. At the center of the given configurations the net electric force is zero because all charges are equal and distances from charges to the center are equal.

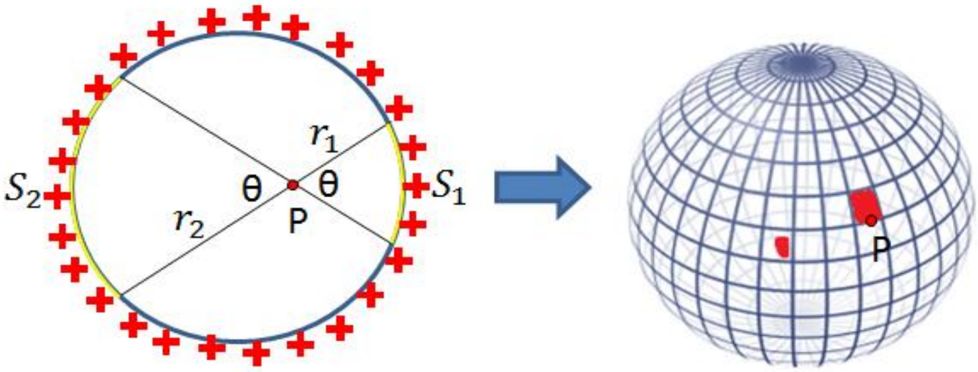

Fig. 2. Drawing analogies between a "charged circle", surfaced charged sphere and volume charged sphere

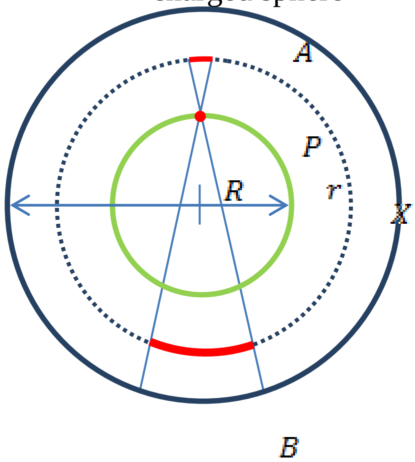

Fig. 3. Layout of three shells of different radii. Inner shell X is colored with green color.

d) Instruments

Two assessment instruments were created in order to compare the effects of our teaching treatments. Indeed, the threat to internal validity of our quasi-experiment was attempted to minimize by administering different instruments within the pretest and posttest contexts (Ary, Jacobs, Sorensen, & Razavieh, 2010). The purpose was to design assessment instruments that measure students' ability to solve qualitative and quantitative problems within the context of electrostatics and particularly Gauss's law. Thus, using knowledge for the purposes of problem solving is closely related to transfer of knowledge to situations that had not been explicitly considered within the teaching treatments (Marzano & Kendall, 2006).

The pretest consisted of five conceptual questions which were adapted from existing literature (Table 2). Barely, question 1 was adapted from the instrument created by Li & Singh (2017). In addition, we adapted questions 2, 3, 4, and 5 from the Conceptual Survey in Electricity and Magnetism (Maloney, O'Kuma, Hieggelke, & Heuvelen, 2001).

The posttest (Basic Understanding of Gauss's law Survey - BUGS) included 12 questions (Table 3). Questions 1 and 9 adapted from *College Physics* by Giambattista, Richardson, & Richardson (2007) and question 6 adapted from *Electromagnetism and structure of matter* by Lopac, Kulišić, Volovšek, & Dananić (1992).

The each of the correct answer was rewarded by one point at pretest as well as at posttest.

Table 2. *A brief description of pretest items.*

Item 1	Direction of electric field at a given point due to two point charges.	Multiple-choice
Item 2	What happens to electric charge that is brought to the surface of a metal sphere?	Multiple-choice
Item 3	How is the direction of electric field at a given point related to the appearance of field lines?	Multiple-choice
Item 4	What happens to magnitude and direction of net force at a given point if we add a third point charge?	Multiple-choice
Item 5	Influence of charge outside the metal sphere on charge located in the sphere's center.	Multiple-choice

Table 3. *A brief description of posttest items.*

Item 1a	What is the magnitude of the electric force on a charged sphere placed in a uniform electric field?	Open-ended
Item 1b	What is the direction of the electric force on a charged sphere placed in a uniform electric field?	Open-ended
Item 2	Comparing the magnitude of electric field at three points inside the uniformly surface charged, hollow cube.	Multiple-choice
Item 3	Predicting direction and magnitude of the net force at the center of a charged, infinite cylindrical conductor.	Multiple-choice
Item 4	Comparing the magnitude of electric field at three points inside a hollow, volume charged spherical shell.	Multiple-choice
Item 5	Can lightning sparks harm a person who is located in a metal cage?	Open-ended
Item 6a	Deriving the electric field expression for a point located at the surface of a volume charged sphere.	Open-ended
Item 6b	Deriving the electric field expression for a point located outside a volume charged sphere.	Open-ended

Item 6c	Deriving the electric field expression for a point located inside a volume charged sphere.	Open-ended
Item 7	Comparing the electric field at three given points for a body that consists of a non-conducting cylindrical shell in whose interior there is a cylindrical conductor.	Multiple-choice
Item 8	Predicting magnitude of electric field at different points of a surface charged metal disc with four symmetrically positioned cavities.	Multiple-choice
Item 9	Determining sign of the charge that is on the inner surface of a spherical shell if within the shell there is a small positively charged sphere.	Open-ended

The reliability of BUGS was calculated as 0.503 based on its Cronbach's alpha. This value can be considered as acceptable (Bowling, 2005, p. 397; McKagan, Perkins & Wieman, 2010). In addition, the average item difficulty index for the posttest was 0.46 which is close to the optimal value (Cohen & Swerdlik, 2009). Furthermore, the difficulty indices of most of the items ranged between the typically recommended boundaries of 0.2 and 0.8 (Kline, 2015). Indeed, there were only two items with difficulty indices outside the above mentioned boundaries - Item1b and Item 6c proved to be very difficult for our students, with difficulty indices of 0.19 and 0.07, respectively.

RESULTS

a) Pretest and posttest scores across subgroups

From Table 4, it can be concluded that students from experimental subgroups performed similarly to students from control subgroups on pretest. The average pretest score was highest for experimental subgroup 1 (EG1) and lowest for experimental subgroup 3 (EG 3). It should be noted that at the posttest, the score in the lowest performing experimental subgroup (EG3) was still higher than the score in the top performing control group (CG1).

Table 4. *Average pretest and posttest scores for experimental subgroups (EG) and control subgroups (CG) are provided. Theoretically, the scale for the pretest ranges from 0 to 5, and for the posttest it ranges from 0 to 12.*

	EG1	EG2	EG3	CG1	CG2	CG3
Pretest	2.0	1.68	1.39	1.97	1.93	1.77
SD	1.19	0.74	1.14	1.01	0.95	1.11
Posttest	6.55	6.52	5.91	5.26	4.93	3.87
SD	1.40	2.25	1.79	2.46	2.03	1.70

Since there can be observe a consistent effect in favor of experimental subgroups, it was decided to collapse the data for individual experimental and control subgroups. Thus, the data only analyzed on order to investigate differences between *two* broad groups – experimental group and control group instead of subsections.

b) Between-group differences in score distributions in the pretest and posttest contexts

Figure 3 shows the distribution of pretest scores in the control and experimental group. In both groups the most prevalent score at the pretest was two out of five points. Although, generally the shapes of the distributions were similar in both groups, there was a slightly higher incidence of low scores (0 or 1 points) in the experimental group. Indeed, the percentage of students who scored 2 points or lower on the pretest were 71% in the control group and 79 % in the experimental group.

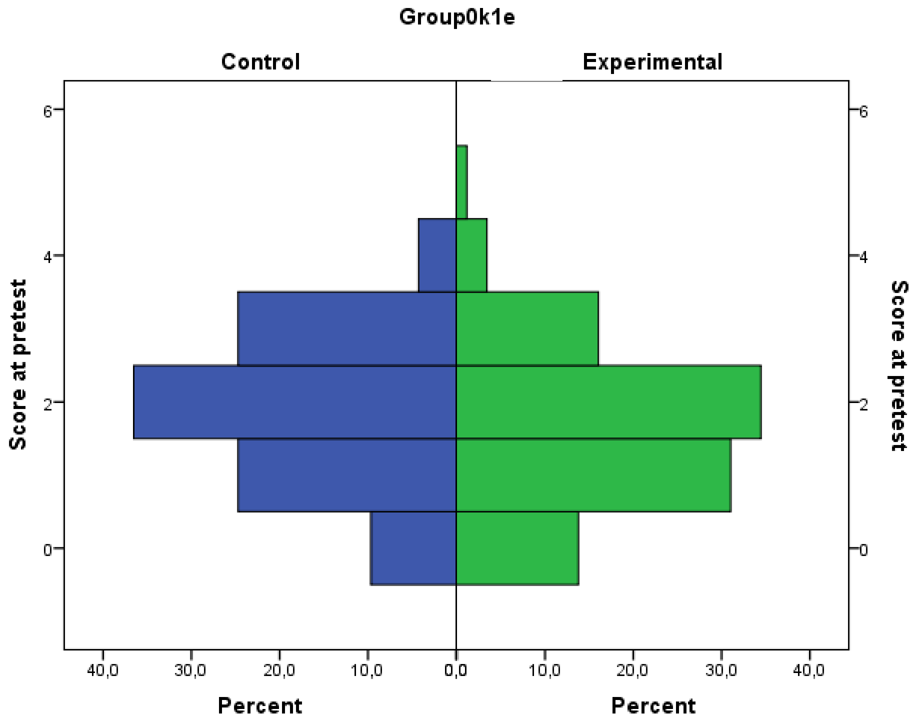

Fig. 3. Distribution of pretest scores in experimental and control group. Theoretically, the pretest scale ranges from 0 to 5.

The distribution of posttest scores for the experimental and control group is presented in Figure 4. From Figure 4, it is evident that the between-group differences at posttest are much more pronounced than the case at pretest.

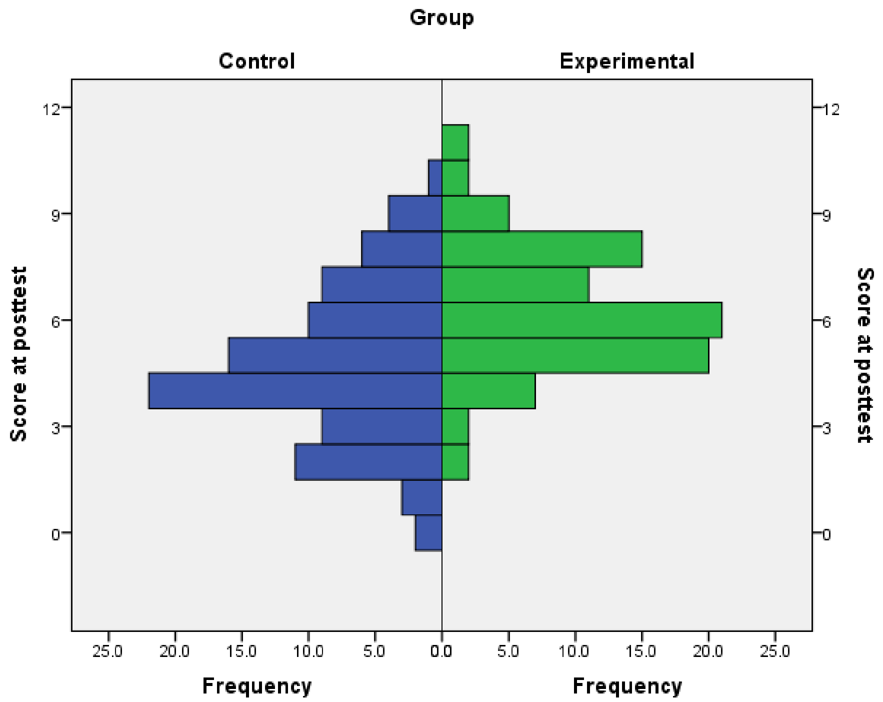

Fig. 4. Distribution of posttest scores in experimental and control group. Theoretically, the posttest scale ranges from 0 to 12.

From Figure 4, it is clear that within the experimental group the share of higher scores (i.e., > 7 points) is larger, and the share of very low scores (i.e., < 4 points) is smaller than in the control group. Indeed, the percentage of students who scored 2 points or lower was 17.2% in the control group at the posttest and 2.3% in the experimental group

Table 5 presents a summarized overview of students' achievement on pretest and posttest.

Table 5. *A summarized overview of average pretest and posttest scores provided for experimental and control groups. Theoretically, the scale for the pretest ranges from 0 to 5, and for the posttest it ranges from 0 to 12.*

	Pretest	Post-test
Control group	1.89	4.70
SD	1.02	2.17
Experimental group	1.68	6.30
SD	1.08	1.83

c) Investigating the significance of the observed between-group differences

The analysis of covariance (ANCOVA) was decided to use for investigating the statistical significance of the observed between-group differences on posttest which allow taking into account between-group differences on pretest (Field, 2009). Before running ANCOVA, the assumption of independence could be showed of the covariate (pretest score) and treatment ("group" as teaching treatment variable) was met ($t(178)=1.36$, $p=.17$). In addition, Q-Q plots showed that normality assumption approximately met for the control and experimental group (Howell, 2013). Since the Levene's test proved to be non-significant ($F(1,178)=2.38$, $p=.12$), there can be concluded that the homogeneity of variance assumption was met. Finally, it was also showed that the homogeneity of regression slopes assumption was not violated – the interaction between covariate and treatment variable proved to be non-significant ($F(1,176)=0.48$, $p=.49$)

ANCOVA results showed that there was a significant effect of teaching treatments on students' posttest scores after controlling for pretest scores ($F(1,177)=32.26$, $p<.001$, partial $\eta^2=.15$). A planned contrast showed that students from the experimental group significantly outperformed their peers from the control group ($t(177)=5.68$, $p<.001$, $r=.39$).

d) Item-level analyses

A summarized overview of between-group differences on individual posttest items is presented in Table 6.

Table 6. *Proportion of correct answers on individual posttest items is provided. Results are provided separately for control group (CG) and experimental group (EG).*

	Item 1a	Item 1b	Item 2	Item 3	Item 4	Item 5	Item 6a	Item 6b	Item 6c	Item 7	Item 8	Item 9
CG	.366	.129	.376	.613	.624	.462	.516	.398	.086	.366	.269	.495
SD	.484	.337	.487	.490	.487	.501	.502	.492	.282	.484	.446	.503
EG	.448	.253	.368	.920	.931	.816	.506	.391	.057	.655	.552	.402
SD	.500	.437	.485	.274	.255	.390	.503	.491	.234	.478	.500	.493

From Table 6 it is evident that experimental group students substantially outperformed than their peers from the control group on 7 out of 12 items and a remarkable difference (9%) in favor of control group students has been detected only on Item 9. In Item 9, students were expected to reason about the sign of the charge on the inner surface of a spherical shell in which is a small, negatively charged sphere. On the other hand, the largest difference in favor of experimental group students was detected for Item 5 (35%) which was supposed to assess students' understanding of the Faraday cage.

In Table 7 and Table 8, the most frequently made errors at pretest and posttest are presented.

Table 7. *Most frequent errors at the pretest.*

	Item 1	Item 2	Item 3	Item 4	Item 5
Pretest (overall)	A (47%)	D (26%)	A (12%)	D (27%)	E (48%)

Table 8. *Most frequent errors at the posttest. Results are provided separately for control group (CG) and experimental group (EG).*

	Item 1a	Item 1b	Item 2	Item 3	Item 4	Item 5
Post-test (CG)	Difficulties with the use of scientific notation (5%) Missing answers (50%)	Electric force direction is always equal to electric field direction (9%)	B (32%)	B (31%)	B (29%)	Students answered that it is possible but did not explain their answer in terms of physics (36%)
Post-test (EG)	Difficulties with use of scientific notation (16%) Missing answers (38%)	Electric force direction is always equal to electric field direction (16%)	B (57%)	B (6%)	B (4%)	Students answered that is possible but did not explain their answer in terms of physics (15%)
	Item 6a	**Item 6b**	**Item 6c**	**Item 7**	**Item 8**	**Item 9**
Post-test (CG)	Electric field is zero at the surface of a volume charged sphere (6%) Missing answers (26%)	Missing answers (26%)	The expression for electric field is the same inside and outside of the volume charged sphere (21%)	C (29%)	C (35%)	A positively charged sphere induces positive charge on the inner surface of the surrounding spherical shell (31%)
Post-test (EG)	Missing answers (24%)	Missing answers (25%)	The expression for electric field is the same inside and outside of the volume charged sphere (29%)	D (19%)	B (36%)	A positively charged sphere induces positive charge on the inner surface of the surrounding spherical shell (38%)

DISCUSSION

a) Overall between-group differences

The average score on the pretest was 1.74 out of 5 points. It should be noted that the pretest questions covered topics that are considered to be important for learning Gauss's law, such as superposition of electric field for different configurations of point charges (items 1 and 4) as well as qualitative ideas about distribution of charge on a conductor (Item 2), electric field lines (Item 3), and the concept of electrostatic shielding (Item 5). All these topics are included within the official high-school curriculum in Croatia which leads us to the conclusion that there is much place for improvement when it comes to developing conceptual understanding about electrostatics at the upper secondary school level in Croatia. The disappointingly low conceptual understanding of electrostatics at pretest could be at least partly accounted for the fact that physics instruction in Croatia typically follows traditional approach at all educational levels (Marušić & Sliško, 2012; Planinic, Ivanjek, & Susac, 2010).

The teaching treatments were only partly successful in promoting development of conceptual understanding about Gauss's law. Indeed, in the control subgroups the average proportion of correct answers at posttest was 39 %, and in experimental subgroups it was 53%. According to results of ANCOVA, the observed between-group difference can be considered as large and statistically significant (Pallant, 2010). Unlike students from the experimental subgroups, students from the control subgroups entered the problem solving session with less developed visual mental models about Gauss's law. Greca & Moreira (2000) suggested that developing of deep conceptual understanding about physics phenomena is often associated with development of corresponding internal visualizations of mechanisms that are at the mere heart of these phenomena. In experimental subgroups, students considered a sequence of increasingly complex configurations of charges and applied the superposition principle with the aim of reasoning about net electric field at arbitrary points. External visualizations were used and the segmenting principle was applied to prevent cognitive overload, i.e. gradually progressed from simple to more complex configurations of charges (Mayer & Moreno, 2003; Mayer, Hegarty, Mayer, & Campbell, 2005; Redish, 2003). Thereby, analogical and extreme case reasoning was used for the purposes of facilitating the transition from reasoning about net electric field inside a circle to reasoning about the net electric field inside a charged sphere. According to findings from earlier researches, usage of analogies and extreme cases helps students to create imaginable and intuitively grounded mental models of physical phenomena (Clement, 1988, 1993; Stephens & Clement, 2009, 2010; Zietsmann & Clement, 1997) which could explain the observed between-group differences.

The very low effectiveness of the traditional practice sessions can be explained based on results of the study by Kim and Pak who found that many students fail to overcome conceptual difficulties even after solving more than 1000 traditional problems (Kim & Pak, 2002). In addition, earlier researches showed that qualitative problems about Gauss's law which were included similar to the ones in BUGS prove to be extremely difficult for students in introductory physics courses (Aubrecht & Raduta, 2005; Guisasola et al., 2008; Maries et al., 2017; Pepper et al., 2010; Singh, 2006; Singh, 2005).

b) Between-group differences at the item-level

In the following lines, the most prominent between-group differences are going to be discussed on individual posttest items.

Item 9 was the only item for which a considerable between-group difference (9%) was detected in favor of control group students. In item 9 students were required to think about a metal spherical shell in center that was a positively charged, small sphere. Indeed, students were asked to determine the sign of the charge at inner surface of the spherical shell. In order to correctly solve item 9, students had to use their knowledge about charging by induction. Considering the fact that in the experimental subgroups, the focus was on reasoning about the net electric field for various configurations of charges, it is no surprise that in item 9 students from the experimental group did not outperform than their peers from the control group. If it is known that none of the two teaching interventions was explicitly focused on the charging by induction concept, then it is concluded that the better performance of the control group students at item 9 can be at least partly explained by their higher initial level of understanding about electrostatics, i.e. by their better results at the pretest.

The most prominent differences were observed for item 3, 4 and 5 in favor of students from experimental group. In items 3 and 4, students were required to think about the electric field at the center of a conducting cylindrical shell and at different points of a volume charged spherical shell, respectively. Within the experimental teaching intervention, students consistently used symmetry arguments to prove that electric field is zero in the center of objects of different shapes which can explain the results for item 3. In addition, students from the experimental group generally learned how to use the superposition principle to get an intuitive insight about electric field at some arbitrary point for a given configuration of charges which probably accounts for the observed between-group differences on item 4. Actually, the students from the control group were trained to predominantly rely on the mathematical formalism when thinking about electrostatics problems which proved to be relatively demanding within the context of item 4. On the other hand, many students from the experimental group probably approached the problem qualitatively by attempting to apply the superposition principle – on the line that extends through A (center) and B (point at outer surface), all charges produce the field of same direction in point B, which suggests that in point B the magnitude of the field is at its maximum. Generally, better achievement of experimental group students on items 3 and 4 is in line with the idea that visually rich models are often more effective than abstract models when it comes to solving qualitative problems (Greca & Moreira, 2000; Mešić, Hajder, Neumann, & Erceg, 2016; Nersessian, 2008). Since the visually rich models are easier activated in authentic contexts (Clement, 2008; Nersessian, 2008), it could also at least partly explain the superiority of experimental group students on item 5 which required the students to judge whether for a boy it would be safe to reside in a metal cage that is being struck by a lightning. It is interesting to note that on a similar item that was situated within a more formal context, students from the control group slightly outperformed their peers from the experimental group. In fact, the most students from the experimental group answered on item 2 that for a surface charged cube the electric field will be zero only in its center.

c) Students' misconceptions about electrical phenomena

First, the students' misconceptions are going to be discussed which observed at the pretest. Since the students from all subgroups were exposed to exactly the same curriculum before taking the pretest, the misconceptions identified at the pretest were similar across all subgroups. So, our discussion of misconceptions in the pretest context will be based on the data gathered across all subgroups.

In item 1 of the pretest, students were showed an equilateral triangle whereby in two vertices of the triangle that there were two point charges of equal magnitude and opposite sign. Students were asked to reason about the magnitude and direction of the net electric field at the third vertex of the triangle. Thereby, even 47% of the students answered that the net electric field has a zero magnitude at the given point. It follows that many students think that the electric fields of two point charges of equal magnitude and opposite sign eliminate each other at all points that are at equal distance from both charges. An alternative explanation is that some students believe that there must be a charge at a given point if the electric field at that point is to be non-zero (Li & Singh, 2017). In other words, some students think that electric field has a zero magnitude at all points in which there is no charge.

In Item 2 of the pretest, students were asked to predict what we will observe a few seconds after adding a small amount of charge to an arbitrary point P on the surface of a hollow, metal sphere. The 26% of the students answered for this item that the most of the charge will remain at point P and a smaller portion will spread over the surface of the sphere. Some of the reasoning can be characterized as reflecting to hybrid knowledge, i.e. knowledge that metal surfaces conduct charge combined with the intuitive belief with this point the largest initial concentration of charge will preserve its "status" even a few seconds after the experiment. This also indicates that many students are not aware of the order of magnitude of the time that has to pass until the establishment of electrostatic equilibrium. Generally, our findings for item 2 are in line with Maloney's (2001) conclusion that many students are confused about the ways in which charge distributes over a conductor.

In item 3, students were shown the electric field lines for a configuration of two point charges of opposite sign, whereby the negative point charge was located to the right side of the positive charge. They were required to reason about magnitude and direction of the net electric field at a point located on one of the field lines approximately above the negative point charge. On this item, 12% of the students answered that the electric field is directed from left to right. One possible explanation is that they overgeneralized their experiences with representations of uniform electric fields between plates of a capacitor where the electric field vector is always directed from the positively to the negatively charged plate. This finding is in line with Raduta's (2005) assertion that students have difficulties to relate electric field lines with the direction of the electric force vector.

On 4^{th} item of the pretest, students were shown three point charges. The charges Q_2 and Q_3 were positive and located on the y-axis whereas charge Q_1 was located on the x-axis and was at equal distance to the other two charges. Students were also provided the information that the force on Q_1 was directed along positive x-axis, and they were asked to predict what will happen with the net force on Q_1 after a new, positive charge Q is added to the x-axis. On this item, 27% of the students had the misconception that adding the charge Q will affect the action of charges Q_2 and Q_3, i.e. they believed that in calculating the net electric force the contributions from different point charges depend on each other. This result supports the assertion that students have many difficulties with the principle of superposition and calculation of the vector sum of the field (Li & Singh, 2017).

In item 5, students were shown an electrically neutral hollow sphere, whereby a positive point charge was located inside the sphere and another positive point charge was located outside the sphere. Students were required to reason about electrical interaction between these two point charges. On this item, 77.8% of the students chose options A or E reflecting to the belief that charge inside the sphere will act with a force on charge outside the sphere and vice versa. Thereby, the option E was the most frequently chosen option and it reflected the belief that the point charges experience electric forces of different magnitude. In the study by Maloney et al. (2001), the same item was correctly answered by only 16% of the students and the most common distracter was also option E.

Finally, students' difficulties are also going to be discussed with solving the posttest (Table VIII).

In Item 1a, students were required to calculate the magnitude of electric force acting on a charged sphere located in a uniform field, whereby information about amount of charge on the sphere and magnitude of electric field has been explicitly provided. Although the most of the students knew the relationship between electric force and electric field, some students proved to have difficulties with using the scientific notation and SI unit prefixes. So, these students failed to correctly calculate the product of amount of charge and magnitude of electric field. University students' difficulties with using exponents and scientific notation had been already reported in other studies (An & Wu, 2012; Heck & Van Gastel, 2006; Shepherd, Selden, & Selden, 2012).

When it comes to the direction of the electric force acting on the sphere (Item 1b), many students answered that is the same at the direction of the given electric field (CG: 9% and EG: 16%), despite the fact that the sphere was negatively charged.

In item 2, students were shown a surface charged cube and asked to compare the electric field at different points inside the cube. The most common error for both student groups (CG: 32% and EG: 57%) was the thought that electric field is zero at the center of the cube, but non-zero at other points inside the surface charged cube. Although it is attempted to provide an intuitive explanation in the experimental group for the fact that field is zero in all points inside a surface charged sphere, it seems that many students failed to transfer that reasoning to different geometrical shapes. An alternative explanation is the conceptual differences between the surface charged and volume charged bodies not been sufficiently discussed within our experimental treatment.

In Item 3, students were asked about the electric field in the center of a positively charged infinite cylindrical conductor. Whereas even 92% of the students from experimental group correctly solved this item, 31% of the students from the control group answered that the electric field at the center of the conductor will have a non-zero magnitude. The result for this item indicates that the experimental teaching treatment succeeded in developing the student habit to consider symmetry arguments when reasoning about the net field at the center of objects. Indeed, superposition principle had been used for finding the net field within the experimental treatment at the center of many geometric configurations of charges (line, square, octagon, circle and sphere).

In Item 4, students were asked to compare the electric fields at different points of a volume charged sphere with spherical cavity. Whereas even 93% of the students from the experimental group correctly solved this item, 29% of the students from the control group recognized that the field at the center of the cavity is zero but at the same time they believed that in the uniformly charged region the field will be the same at all points. In other words, it seems that some students think that a uniformly charged volume implicates a uniform field within that volume.

In Item 5, students were shown a photo depicting a boy in a metal cage being struck by lightning and asked to discuss whether the photo is genuine. Many students from the control (36%) and experimental group (15%) did not thoroughly explain their answer in terms of physics. Actually, many students did not provide a verbal explanation at all, particularly in the control group. It seems that mental models, which are mostly based on mathematical formalism, are relatively ineffective when it comes to generating rich, qualitative explanations which is in line with the findings by Greca and Moreira (1997).

In items 6a, 6b and 6c, students were required to derive the expression for electric field at three characteristic points of a volume charged sphere. Thereby, students had extreme difficulties with solving the volume integral and confused the expression for electric field with the Coulomb's law. Generally, earlier researches showed that students are having difficulties with applying integral calculus in the context of Gauss's law and electromagnetics in general (Bollen et al., 2015; Pepper et al., 2012).

In item 7, students were required to compare the electric field for a complex body consisting of a volume charged cylindrical shell in whose interior was a surface charged cylinder. In the control group, 29% of the students answered that the magnitude of the field is the highest on the central axis of the surface charged cylinder, and that it decreases as the radial distance from the center increases, becoming zero at the surface of the volume charged cylindrical shell. In other words, many students from the control group believed that the electric field at the surface of a non-conducting object has a zero magnitude. In the experimental group 19% of the students chose distracter D. They correctly recognized that the field inside the conductor is zero, but at the same time they did not realize that the electric field across the volume charged shell increases as we approach its surface. It is interesting to note that students from the experimental group were significantly less successful on item 7 than on the conceptually similar item 4. A possible explanation is that an intuitive approach to reasoning about the field of a volume charged *sphere* considered within the teaching treatment and for students it was easier to transfer the knowledge to the context of a thick spherical shell (item 4) than to the context of a cylindrical shell (Item 7).

In item 8, students were shown a surface charged metal disc with four symmetrically placed cavities. Students were required to reason about the electric field at the center of the disc as well as about the field in the cavities. Although the metal disc is thin, it is still a three-dimensional object which implicates that the field in its interior has to be zero and the field inside the cavities also has to be zero. In the experimental group, 36% of the students recognized that the electric field at the center of the disc is zero but at the same time they believed the fields at the center of the cavities to be non-zero. On the other hand, 35% of the students from the control group again did not recognize that the field inside a conductor has to be zero for the case of electrostatic equilibrium and they had the misconception that the electric field inside cavities should have a negative sign. The results for item 8 show that students from experimental group consistently outperformed their peers from the control group when it comes to reasoning about the electric field at the center of various objects.

In item 9, students were required to determine the sign of the charge at inner surface of a conducting spherical shell that its interior was a positively charged, smaller sphere. Students from the control group proved to be more successful in applying the concept of charging by induction for solving this qualitative problem.

d) Limitations of the study

Although the experimental treatment proved to be significantly more effective than the traditional treatment, results of the posttest indicate that none of the two teaching treatments succeeded to develop a high level of ability for solving qualitative and quantitative problems about Gauss's law in students. Particularly the performance on quantitative problems was very low in both groups.

It seems that there is much place for improvement within the experimental teaching intervention when it comes to differentiating the contexts of volume charged and surface charged bodies. Further, it seems that students need additional help to transfer their understanding about the field generated by charged spheres to fields generated by objects of different shapes.

When interpreting the findings from our study, one should take into account that these findings were obtained for an introductory physics course in which lectures follow a typically traditional format.

SUMMARY AND CONCLUSION

In this study, it was aimed to investigate whether enriching practice sessions with analogies and extreme cases can facilitate development of understanding about Gauss's law in university students. Indeed, it was aimed to find out whether reasoning about principle of superposition of electric field for relatively simple configurations of point charges can be effectively transferred to more complex geometric configurations and continuous distribution of charge.

The most important conclusions from our study are as follows:

- A step-by-step application of the superposition principle for increasingly complex configurations of charges facilitates the development of conceptual understanding about the Gauss's law.
- Combining analogical and extreme case reasoning can help to effectively relate the electric fields generated by a uniformly charged circle (very thin ring), a surface charged sphere and a volume charged sphere.
- Visually rich, analogical mental models are less inert in comparison with the mental models that predominantly include mathematical representations. They are also more effective for solving qualitative problems, particularly in contexts that are perceived by the students to be very different to contexts they explicitly encountered within instruction.

Suggestions

For purposes of developing students' understanding about Gauss's law, physics teachers are advised to invest additional efforts in relating the Gauss's law to the principle of superposition of electric field vectors. Thereby, it is recommended to gradually lead the students from reasoning about superposition of electric field vectors for a one-dimensional configuration of point charges to reasoning about the net electric field that results from a three-dimensional, continuous distribution of charges. The transition from discrete to continuous distributions of charges can be effectively accomplished by introducing a uniformly charged, very thin ring as an extreme case of a polygon (in which vertices are point charges) when the number of polygon's edges tends to infinity. Next, the electric field of a uniformly charged ring can be used as an analogical anchor for reasoning about the electric field of a surface charged spherical shell and the volume charged sphere can be introduced as an extreme case of the surface charged shell. In order to reduce the possibility of conceptual confusion, it is advisable to explicitly discuss with the students about similarities and differences in electric fields of the thin ring, surface charged spherical shell and volume charged sphere.

Although the presented teaching approach proved to be relatively effective in developing basic conceptual understanding about Gauss's law, there is still room for its improvement.

In future research, it would be useful to develop and evaluate strategies for helping the students to transfer their reasoning about charged spheres to charged objects of different shapes. Furthermore, it would be potentially interesting to explore whether students' understanding about the electric field inside volume charged spheres can be improved by introducing an analogy with the gravitational field inside the Earth.

Generally, the effectiveness of teaching about Gauss's law by combining analogies and extreme cases could be additionally explored through implementation of a mixed-research study which would also include student interviews.

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