

The RME Principles on Geometry Learning with Focus of Transformation Reasoning through Exploration on Malay Woven Motif

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ABSTRACT

The use of concrete objects in a real-life situation is one of the effective ways for teachers to teach and students to explore and learn transformation geometry. The present study, which used a local wisdom or learning context (called Malay woven motif of Kepulauan Riau), focused on developing the reasoning of transformation geometry with Realistic Mathematics Education (RME) approach. The sample of this two cycles of design research consisted of grade 4 students from a state elementary school of 001 Toapaya, Bintan Regency. The questions ‘How does this learning approach illustrate some characteristics?’ and ‘How well does it perform?’ need to be responded. Hence, the RME principles (i.e., activity, reality, level, intertwinement, interactivity, and¹ guidance) in the implementation of learning have been re-formulated over the years. This study showed that learning geometry through transformation reasoning with all RME principles helped students actively learn mathematics. The context of Malay woven motif brings real and meaningful problems for students to enable the process of mathematization. The learned mathematical content intertwined with other mathematical contents allows learning process to emphasize individual, social, and teacher guide activities.

Keywords: Malay woven motif, reasoning, RME principle, transformation geometry

INTRODUCTION

Geometry, which is limited to two dimensions, is called plane geometry. One of the concepts in plane geometry is transformation, which maps a geometry object on a plane from one position to another with certain rules (Hardiyanti, 2015). The mapping character emerges two types of the transformation, i.e., isometric and nonisometric. Stillwell (2004) states that the isometric transformation of an object from one location to another shows a distance between two points



on the object. Hence, the size of the object will be the same before and after the transformation. This study focused on the geometry of isometric transformations (e.g., reflection, translation, and rotation).

Due to its dynamic nature, transformation is considered as a crucial knowledge in geometry. Understanding transformation geometry includes spatial ability, geometric reasoning, and mathematical proof (Edwards, 1997). At least, this explains the reason why the school mathematics or mathematics curriculum contains the transformation geometry. The National Council of Teachers of Mathematics (NCTM) (2000) states that learning transformation geometry should be taught from kindergarten to high school level, so that students are able to use the transformation geometry to analyze mathematical situations. This is in line with the experts' views suggesting that learning transformation geometry provides an opportunity for students to explore such mathematical concepts as symmetry, congruence, functionality, equivalence and so forth. Further, the transformation geometry involves diverse disciplines in the development of reasoning (Hollebrands, 2003; Panorkou, Maloney, Confrey, & Platt, 2014). Reasoning, which means decision-making and a logical process, is related to the transformation geometry (Febrian & Perdana, 2018). For this reason, reasoning is on the focus of this study.

Referring to the KTSP and the 2013 curriculum in Indonesia, the geometry, especially transformation, is firstly introduced to grade 3 and 4 students. However, teaching transformation geometry with knowledge transfer method makes its effectiveness weak (Febrian & Perdana, 2017). In addition, a lack of the use of concrete objects makes this topic abstract for students and results in reasoning weaknesses.

Conducting a new study on the material of transformation geometry supports reasoning ability. This research used the Indonesian version of Realistic Mathematics Education (RME) approach to set learning. The RME is a domain-specific instructional theory handling mathematics as a subject, children's mathematics learning and how to teach mathematics (Van den Heuvel-Panhuizen, 1996). Although "realistics" in this approach means "real-world" situations in the RME, the word itself has a broader meaning. It depicts the problem that students can imagine (Van den Heuvel-Panhuizen & Drijvers, 2014). Basically, the RME stresses an ideal mathematics learning by doing the mathematics itself. Freudenthal sees mathematics as a human activity. In the learning process, students are expected to re-discover the studied mathematical concepts. Thus, the emphasis in this learning is the process.

RME, as a learning approach, has several principles reformulated by Treffers (Van den Heuvel-Panhuizen & Drijvers, 2014; Van den Heuvel-Panhuizen & Wijers, 2005): (1) activity principle emphasizing that mathematics is learned by doing mathematics itself or known as mathematization to develop concepts and mathematical understanding; (2) reality principle assuming that reality is not only discovered after studying mathematics in solving everyday problems, but also considered as a learning resource and context; (3) level principle explaining that students go through different levels of understanding when learning mathematics in context-related solutions to find related concepts and strategies; (4) intertwinement principle having the meaning that content domains in mathematics cannot be perceived as stands, but are interconnected with each other; (5) interaction principle explaining that learning mathematics is not merely individual activity but also social activity through interaction mechanism during learning; and (6) guidance principle emphasizing teacher's active participation in creating student learning and learning programs to achieve understanding.

Finally, Khusniati, Parmin, and Sudarmin (2017) state that a meaningful and contextual observation arises from student's immediate environment, especially a cultural and local wisdom content. In this point of view, this research used Malay woven motif of Kepulaun Riau as an object to explore and learn. The use of this object may facilitate children's natural spatial senses. The selected Malay motif is called *itik pulang petang*. In this study, students learned the arrangement of *itik pulang petang* in the assignments to build their transformation geometric

reasoning with the RME approach. There are two approaches to understanding the transformation geometry, namely graphical and algebraic ones (Mashingaidze, 2012). This study, which had a focus on graphical reasoning, presented a discussion of the RME principles of in learning transformation geometry reasoning through the exploration of woven Malay motif of Kepulauan Riau, Indonesia.

METHOD

The general objective of this research was to facilitate the development of geometrical reasoning of students' isometric transformation through the exploration of Malay woven motif of Kepulauan Riau. The subject is introduced to grader 4 students at the state elementary school of 001 Toapaya, Bintan Regency, Kepulauan Riau. For this purpose, a design research approach was used: preparing for the experiment; the design experiment; and retrospective analysis. The study consisted of two cycles: preliminary teaching experiment/pilot study (cycle I) with a small-student group and teaching experiment (cycle II) with one class. This paper only provided the results of teaching experiments on the Malay woven motif. The RME principles in the lesson was carried out and presented in this paper. The questions 'How were the RME principles conducted?' and 'How well did the learning perform?' were then open for data analysis. These aspects were critically appraised in the current study.

In preparation of cycle II, hypothetical learning trajectory (HLT) or learning trajectory consisted of: learning objectives, mathematical activities, and conjectured student thinking processes. The presented HLT was updated after the implementation of the first cycle. The HLT for the second cycle of the research is presented in the 'results and discussion' section.

The learning activities were based on the Indonesian version of Realistic Mathematics Education (RME) approach by using the context of Malay woven motifs of *itik pulang petang*. Subjects in the teaching experiment cycle were at grade 4A. The collected data included activity data and student learning, as well as their thinking processes through observation techniques. That is, video recorders and interview techniques were used to explore their thinking abilities. The next data were a series of the student assignments. All data made the current study possible to respond critical appraise on how the RME principles took stand in the learning process and how well the learning performed.

To facilitate the researchers to do a retrospective analysis, all collected data were analyzed by triangulation technique. Segments in videos depicting the students' thinking processes through interviews or discussions were expressed in conversational transcripts and presented in the results. Descriptive qualitative techniques were employed to provide the RME principles in learning transformation geometry reasoning using the context of Malay woven motif of Kepulauan Riau.

RESULTS AND DISCUSSION

The learning design (consisting of five activities) with the Malay woven motif emphasized on the reasoning of the transformation geometry. Prior to the outcomes and discussions related to the RME principles, the HLT for learning cycle 2 is presented to illustrate its implementation.

Table 1. HLT for learning cycle 2

Mathematical Activities	Tasks	Conjecture of Students' Thinking	A Snapshot of the Activity
Reasoning about the arrangement difference in the motifs of the three varied patterns (Activity 1)	Students observe three varied arrangements of the motif displayed in the gallery-- 'reflection, rotation, and translation' techniques. Students report the result(s) in written and verbal expressions	<ul style="list-style-type: none"> • Students produce informal knowledge such as unit repetition and isometric through object size • Students produce informal terms of transformation based on their observations and comparative activities • Summarizes the differences between the three varied motif arrangements from the technique. 	
With the informal knowledge of activity 1, the students predict and draw the transformation arrangement using a piece of motif as the unit (Activity 2-3)	The activity is called " <i>with a piece of motif, let's think about the drawing technique</i> ". Students are asked to remember the arrangements of three varied motifs. The three variations are retained. The students are asked to play the weaving maker, write a strategy to make variations in the worksheet, and re-create the variation arrangement process. Students are given one piece of <i>itik pulang petang</i> .	<ul style="list-style-type: none"> • Students review visual reasoning of the motif arrangement by producing informal knowledge of transformation • Students predict the technique on making varied motifs by help of one motif • Students perform a hands-on activity on a piece of the motif by moving, rotating, shifting, reversing to adjust the forms of variations in the remembered arrangement • The idea of a constant factor in translation and rotation 	 
Moving motifs on grid media with certain transformations	Students are asked to use the grid media to transform the motif in a triangle	<ul style="list-style-type: none"> • Using the grid to facilitate the transformation • Using certain points and lines as a reference for transformation 	

Investigating the point and orientation of motif image if a particular transformation is made	Students investigate point location and motif orientation that are undergone a certain transformation	<ul style="list-style-type: none"> • Reasoning to use a reference point to find the positions of other points • Using points and lines in determining transformation types • Revealing the image(s) as the result(s) of one or more times of a particular transformation (a combination of transformations)
(Activity 4 a & b)	Students are asked to find the type of transformation if its image is given.	

Exploring the translations, rotations, and reflections on the grid vis-à-vis a motif	Students are required to use the grid and piece of motif to explore transformations by using via their own creativities	<ul style="list-style-type: none"> • Students use grids for transformation media • Students think about orientation, reference point, constant factor in describing transformation object
(Activity 5)		



Furthermore, the HLT had been designed before the implementation of learning in the second cycle was compared to the actual learning path (ALT). Results appeared no difference between the HLT and ALT. In this study, students showed all mathematical activities and thinking processes hypothesized by the researchers. The analysis and discussion of the RME principles in learning transformation geometry through Malay woven motif of Kepulauan Riau are presented as follows:

a) Activity Principle

Activity 1

The purpose of learning in the first activity was to reason visually about the concept of reflection, translation, and rotation and their characters through observation on variations of motif patterns. The Malay woven gallery (as the context) showed three different patterns of a single motif of itik pulang petang. The variations were arranged with 'translation, reflection, and rotation' techniques. Through the observation of differences in the three pattern variations, the students were asked to report the result(s) in written and verbal expressions. In this activity, the students tried to make sense the arrangement of motif in each variation. The students produced informal knowledge connected to the ideas of unit repetition and constant unit size. Through this idea, students built the informal kind of isometrical transformation geometry.

In view of Febrian and Perdana (2018), some informal terms representing translations, e.g., 'arranged in a row', 'slided/shifted', and 'arranged in March,' appeared during the learning process. Further, the informal terms describing reflections were 'made opposite', 'opposite direction', and 'pairing'. Rotation was represented by the phrase 'rotated', 'made circular', 'rounded', and 'around'. The arrangement setting and sequencing of the pattern variation made the students visually aware of the differences in the single motif arrangement in each variation. Hence, activity 1 purposed to persuade the students about the unit repetition and transformations at the informal level.

Through the first activity, the students were engaged in analyzing the situation set within the task. They actively participated in making sense the arrangement differences observable. Overall, the students developed some insights to understand the case, which would be used for the next tasks.

Activity 2

In this activity called "*with a piece of motif, let's think about the drawing technique*", students were asked to remember the motif arrangement in variations of patterns 1, 2, and 3. The display of the variation of the pattern was retained. The students were asked to fill in the sheet to predict the 'making pattern variation' technique for the single motif of *itik pulang petang*.

In view of Febrian and Perdana (2018), some of the students' responses included the variation of pattern 1 by shifting a piece of motif continuously in a certain direction and repeatedly several lines. The students were unaware of the constant factor in shifting. Meantime, the question for the variation of pattern 2 was responded more tactfully. The students generally explored a piece of the motif by rotating it to its midpoint; reversed and shifted. They did it to force the motif to be opposite. The student decided to make a pair of motif facing each other. One of the drawings was made by reversing the motif. The students did a tracing technique in motif propagation to build variations. Meanwhile, the 3rd pattern variation was formed by rotating a piece of the motif. However, they were unaware of the rotary axis or rotation reference point in the process. Activity 3 was continued with the same context.

It can be concluded that the students developed their own ways to understand the arrangement of the motif. However, reasoning in this activity was more observable in 'action' way instead of verbal one.

Activity 3

To follow-up the second activity, the third activity was carried out. The students played an artisan role and went back to the artisan process to come up with three motif arrangement variations. Students drew a variety of patterns 1, 2, and 3 from a piece of motif provided with grid paper media, pins, threads, and styrofoam, as well as drawing tools such as pens and markers. Febrian and Perdana (2018) entailed such strategies as: reversed object to make symmetrical figure for the idea of different orientation; preserved distance in case of translation; constant factor in rotation turning around the distance and angle in a very informal way of verbal reasoning by using informal terms.

The students approached the task by fully using the single motif to trace back the artisan's arrangement. It is clear that students tried to make sense the situation and worked the problem themselves to finally figure out the answer.

Activity 4

This activity used the context of the aforementioned motif in the printed grid medium. The activity recruited class discussion. There was the motif of *itik pulang petang* in a triangle. The location or position of the motif resulted in its transformations. Printed grid media and pins to put motifs on the media were used. The purpose of this activity was to enable students to recognize transformations as displacements of transformed starting points, line and point of reference, orientation, constant factor, and transformation composition. In the first case, the motif was located in the lower left corner of the printed grid medium. The triangle corner point was marked with different color pins. Then, another pin was located in another area in the printed grid media. The student task needed to find other two point locations in order for dissipating transformation to the motif.

According to Febrian and Perdana (2018), the students could think about finding other two point locations so that the case was transitive. Other students might even indicate that such displacement might occur as more than one shift was made.

The students were asked to define the type of transformation that the motif position in the initial triangle was reversed (see Table 1, activity 4). Some students could find the location of other points, so that they could define their transformation appropriately. It can be concluded that the students understood the lines and points of reference, orientation, constant factor, and transformation composition.

Activity 5

In this activity, the students explored to move the motifs in triangles through translation, reflection, and rotation on the printed grid media. A printed grid media and stationery set handed each group out to conduct exploration. The activity indicated that the students comprehended translations, rotations, and reflection as the form techniques to move the initial object to another position on the printed grid media. From these results, the students also described their understanding of the orientation of transformed objects, reference points, and isometric transformation geometry.

From all activities, it can be deduced that the students actively participated in making sense the case within the activity. They used their informal ways to tackle the problem-solving strategy. That is, this learning design promoted the first principle of the RME in a very supportive way. Further, they actively took part in the educational process, and developed all mathematical tools and insights instead of passive receivers of ready-made mathematics (Van den Heuvel-Panhuizen & Wijers, 2005). In other words, this learning design was effective in promoting the students to do mathematical process while trying to solve the problem.

b) Reality Principle

In the learning design, the students were presented with the context of the Malay woven motif throughout the variations of arrangement provoking discussion between students' various assignments. The observations revealed that the use of the Malay woven motifs enriched the concept of transformation geometry for students, and enabled them to organize the concepts gradually. This started from building initial reasoning about the composition of the motif, unit repetition, various forms of transformation and concepts accompanying such as factor constant, distance, angle, and reference point. The composition through a series of assignments was designed in such a way. The learning experience with the motif led to a meaningful learning process and helped the students to build their own understanding of transformation geometry even at an informal level. Thus, the use of motif (as a context) acted as a realistic learning resource and student perception. Their generated knowledge can also be used for other situations. This learning thus illustrates the attainment of the principle of the reality on the RME.

c) Level Principle

In this learning design, the five activities using woven motif as a learning context facilitated students to develop the transformation geometric reasoning. Early development of informal visual reasoning, which is related to the transformation geometry through the motif approach, used informal terms and observations through the idea of unit repetition. Knowledge derived from observation was used to evolve such important ideas as reference points, constant factors, and the orientation of transformed objects through the reasoning process of the use of the object. At the end of the observation, the students used these reasoning skills to build the transformation reasoning activity over the grid medium. The results showed that the students were very flexible for performing transformations above the grid and determining the elements asked in the assignment.

These results indicated that the learning design improved several understanding levels of various transformation geometries through a series of activities using the Malay woven motif of Kepulauan Riau. Thus, the instructional design illustrates the third principle of the RME.

d) Intertwinement Principle

Observations from the learning process, especially on activities 2 and 3, produced many informal ideas through transformation reasoning. Mathematical activity helped the students to predict how the arrangement of the motif in three variations was formed by the weaving artist. Writing in the worksheet and verbal reasoning during the hands-on activity showed that the students associated the concept of transformation with those of distance in 'translation and rotation,' circumference in 'rotation', angle in 'rotation', and symmetry in 'reflection.' At the next level, transformation was successfully connected to the concept of composition in activities 4 and 5. The students informally produced all attributions and indicated an association process between several interrelated concepts and transformation. Domains in mathematics are not data separated from each other (Van den Heuvel-Panhuizen & Wijers, 2005). Thus, the principle of the intertwinement-related RME is reflected in this lesson.

e) Interaction Principle

The learning design was structured by highlighting the students' aspects as social learners. The observations from the learning activities (from activity 1 to activity 5) illuminated that the students jointly built their gradual understanding through the processes of discussion, questioning, and negotiating other views and reflection. Because the RME consider the learning of mathematics as a social activity (Van den Heuvel-Panhuizen & Wijers, 2005), the learning design accommodates the student interaction and raises the principle of interaction successfully.

f) Guidance Principle

The observations from the learning process denoted that the created interaction was not only between the students, but also between teachers and students. Discussion was dominant for the setting of learning through question and answer techniques. In instructional design, teacher acts as a facilitator to equip the students with the necessary reasoning and conceptual discoveries in transformation geometry. In this activity, the teacher asked a lot of questions to trigger their students' further thinking skills of the reasoning. For example; the teacher asked the questions "What are the possible positions of other two vertices if this motif above the grid is exposed to any transformation or a single vertex is placed in this section?", "What do you need to note for this rotation to produce a similar arrangement with variations of the given motive?" and so on. Hence, these questions were very powerful to lure the students to build their understanding of transformation. Thus, teachers are very dominant in guiding students. This is very much in line with the last RME principle of guidance.

CONCLUSION

This study showed that learning the transformation geometry with reasoning fulfilled all principles of the RME. The learning design enabled the students to actively learn some important concepts of the transformation geometry by doing mathematics. The context of the Malay woven motif brought real and meaningful problems for the students to organize the students' conceptual understanding of the transformation geometry. This study illustrated that mathematical content knowledge through the learning process emphasized individual, social and teacher-guide activities.

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